Coulomb's law of electrostatics

The force on a test charge Q due to a single point charge q which is at rest at a distance r away from Q, is given by Coulomb's law:

$$\boxed{\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{r^2}\hat{r}} \quad \text{--- (1)}$$

The Constant ϵ_0 is called the "permittivity of free space." Monday 20

In SI units, where force is in Newtons (N), distance in meters (m) and charge in Coulomb (C)

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

Hence, Coulomb's law states that the force is proportional to the product of the charges and inversely proportional to the square of the separation distance.

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As, $\vec{\mathscr{r}}$ is the separation vector from $\vec{r}'$ (the location of q) to $\vec{r}$ (the location of Q):

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' \quad \text{———} \quad (2)$$

$\mathscr{r}$ is its magnitude and $\hat{\mathscr{r}}$ is its direction

The force points along the line from q to Q; it is repulsive if q and Q have the same sign and attractive if their signs are opposite.

Electric Field Intensity

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If we have several point charges $q_1, q_2, \ldots, q_n$, at distances $\mathscr{r}_1, \mathscr{r}_2, \ldots \mathscr{r}_n$ from Q, the total force on Q is evidently.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 \cdot Q}{\mathscr{r}_1^2}\hat{\mathscr{r}}_1 + \frac{q_2 \cdot Q}{\mathscr{r}_2^2}\cdot\hat{\mathscr{r}}_2 + \cdots\right)$$

or,

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{q_1}{\mathscr{r}_1^2}\cdot\hat{\mathscr{r}}_1 + \frac{q_2}{\mathscr{r}_2^2}\cdot\hat{\mathscr{r}}_2 + \cdots\right)$$

$$\boxed{\vec{F} = Q\vec{E}} \quad \text{——— (3)}$$

Where,

$$\vec{E}(\vec{r}) \equiv \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2}\hat{r}_i \quad \text{——— (4)}$$

$\vec{E}$ is called the electric field of the source charges. It is a function of position ($\vec{r}$), because the separation vectors $\vec{r}_i$ depend on the location of the field point P. (fig. 1)

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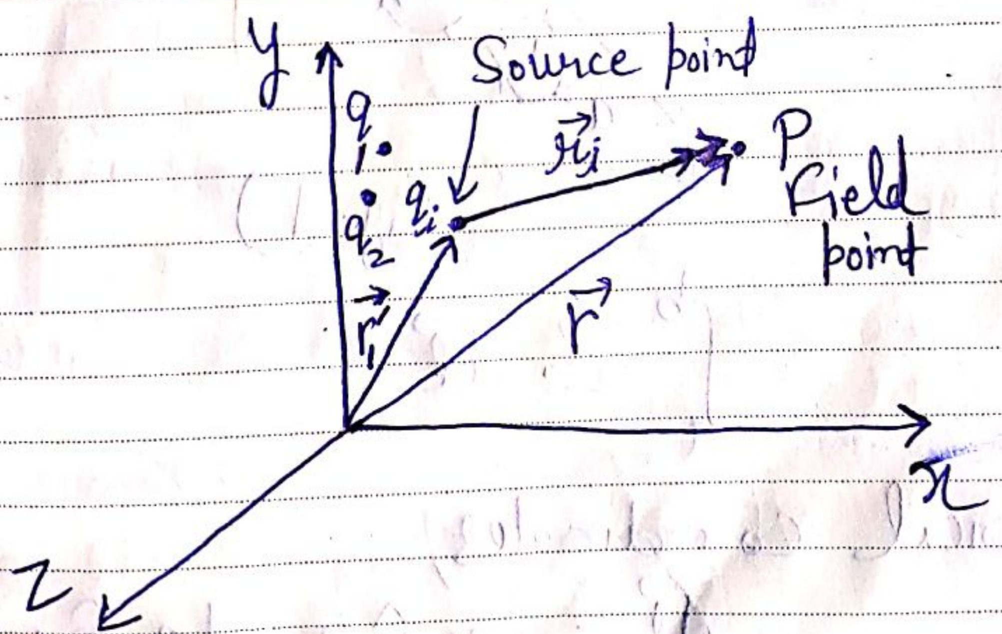


The electric field is a vector quantity that varies from point to point and is determined by the

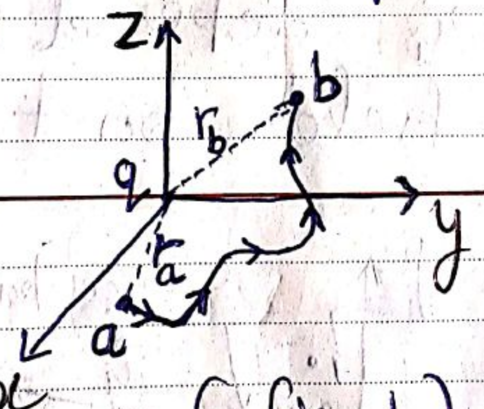
Configurations of source charges.

Curl of $\vec{E}$

By the def of electric field,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \hat{r}$$

first, we calculate the line integral of this field from some point $\vec{a}$ to some other point $\vec{b}$ (fig. 1)



(fig. 1)

$$\int_a^b \vec{E} \cdot \vec{dl}$$

In spherical co-ordinates,

$$\vec{dl} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi},$$

so,

$$\vec{E} \cdot \vec{dl} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot dr$$

Therefore,

$$\int_a^b \vec{E} \cdot d\vec{l} = \frac{1}{4\pi\epsilon_0} \int_a^b \frac{q}{r^2} \cdot dr$$

$$= -\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \Big|_{r_a}^{r_b}$$

$$\int_a^b \vec{E} \cdot d\vec{l} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_a} - \frac{q}{r_b} \right) \quad \text{——— (1)}$$

where, r_a is the distance from the origin to the point $\vec{a}$ and r_b is the distance to $\vec{b}$". The integral around a closed path is zero (for then $r_a = r_b$)

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$$\boxed{\oint \vec{E} \cdot d\vec{l} = 0} \quad \text{——— (2)}$$

and hence, applying Stokes' theorem

$$\boxed{\vec{\nabla} \times \vec{E} = 0} \quad \text{——— (3)}$$

eqs: (2) and (3) are only for the field of a single point charge at the origin.